Technical Comments

Comments on "A Note on the General Solution of the Two-Dimensional Linear Elasticity Problem in Polar Coordinates"

C. W. Bert*
University of Oklahoma, Norman, Okla.

In a recent technical note, W. Z. Sadeh[1] claimed to have found four new solutions to the biharmonic equation. However, these solutions are not new. For example, the first three of them can be found in Ref. 2, and the fourth can easily be obtained from the third by a simple 90° rotation of the coordinate axes. Also, all four were given in Ref. 3.

There appear to be three kinds of solutions of the biharmonic compatibility equation: 1) those having periodic stress components and periodic displacements, 2) those having periodic stress components and nonperiodic displacements (called Volterra-type dislocations[4] if the origin lies within the body), and 3) those having nonperiodic stress components and nonperiodic displacements.

All three types were included in the stress functions in Refs. 2 and 3, whereas Ref. 5 included only the first two types. A derivation of the complete form for two-dimensional stress functions having periodic stress components was presented in Ref. 6.

Nonperiodic stress components have been discussed in detail in Refs. 7–9. As mentioned in Ref. 7, the difference between the Volterra-dislocation type of solution (type 2) and type 3 is that the former is associated with purely rigid-body displacements whereas the latter is associated with elastic displacements. Reference 8 discussed type 3 solutions under the condition that the traction on a stationary surface of discontinuity be continuous. It also pointed out that in the type 3 solution, the location and shape of the dislocational barrier must be specified. Reference 9 discussed type 3 solutions in which even the surface tractions were not required to be continuous.

It is interesting to note that Ref. 2 presented some solutions of the biharmonic equation which were not included in any of the other references. Recently, Ref. 10 has presented couple-stress-function solutions, which, in two-dimensional couple-stress elasticity, couple with all of the solutions of the biharmonic equation given in Ref. 2.

References

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Received February 24, 1967.

* Professor, School of Aerospace and Mechanical Engineering. Associate Fellow AIAA.

Comment on "A Note on the General Solution of the Two-Dimensional Linear Elasticity Problem in Polar Coordinates"

B. I. Hyman*
The George Washington University, Washington, D.C.

The author's[1] inference that he has obtained the general solution to the biharmonic equation is misleading. It is easy to show[2] that the general solution to $\nabla^4\Phi = 0$ is

$$\Phi = f_1(x + iy) + f_2(x - iy) + xf_3(x + iy) + xf_4(x - iy)$$

or, in polar coordinates,

$$\Phi = f_1\,(re^{i\theta}) + f_2\,(re^{-i\theta}) + r\cos\theta f_3\,(re^{i\theta}) + r\cos\theta f_4(re^{-i\theta})$$

where f_1, f_2, f_3, and f_4 are arbitrary functions of their arguments. It is apparent from the discussion by Timoshenko,[3] that the only terms he included in his so-called "general solution" are terms which represent solutions of known physical significance. In fact, Timoshenko includes in his expression the term $d_0r^2\theta$ which does not appear in Michell's[4] original work but which does represent the stress function for a known problem. Neither Timoshenko nor Michell indicate a mathematical procedure for arriving at their results.

A separation of variables solution to the biharmonic equation is presented in Ref. 5 where the solution is assumed to be of the form

$$\Phi = F(r)G(\theta)$$

with $G(\theta)$ represented by either $\sin n\theta$ or $\cos n\theta$. Introducing a new variable $t = \ln r$, and assuming that

$$F = Ke^{\lambda t}$$

Received March 22, 1967.

* Assistant Professor of Applied Science, Engineering Mechanics Dept. Member AIAA.